National Centre for Excellence in the Teaching of Mathematics

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Mastery approaches to mathematics and the new national curriculum

'Mastery' in high performing countries

The content and principles underpinning the 2014 mathematics curriculum reflect those found in high performing education systems internationally, particularly those of east and south-east Asian countries such as Singapore, Japan, South Korea and China. The OECD suggests that by age 15 students from these countries are on average up to three years ahead in maths compared to 15 year olds in England[1]. What underpins this success is the far higher proportion of pupils reaching a high standard and the relatively small gaps in attainment between pupils in comparison to England.

Though there are many differences between the education systems of England and those of east and south-east Asia, we can learn from the 'mastery' approach to teaching commonly followed in these countries. Certain principles and features characterise this approach:

- Teachers reinforce an expectation that all pupils are capable of achieving high standards in mathematics.

- The large majority of pupils progress through the curriculum content at the same pace. Differentiation is achieved by emphasising deep knowledge and through individual support and intervention.

- Teaching is underpinned by methodical curriculum design and supported by carefully crafted lessons and resources to foster deep conceptual and procedural knowledge.

- Practice and consolidation play a central role. Carefully designed variation within this builds fluency and understanding of underlying mathematical concepts in tandem.

- Teachers use precise questioning in class to test conceptual and procedural knowledge, and assess pupils regularly to identify those requiring intervention so that all pupils keep up.

The intention of these approaches is to provide all children with full access to the curriculum, enabling them to achieve confidence and competence – 'mastery' – in mathematics, rather than many failing to develop the maths skills they need for the future.

[1] PISA 2012 http://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf

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Curriculum changes

The 2014 national curriculum for mathematics has been designed to raise standards in maths, with the aim that the large majority of pupils will achieve mastery of the subject. Mathematics programmes of study state that:

- *All pupils should become fluent in the fundamentals of mathematics, including through varied and frequent practice, so that pupils develop conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems.*

- *The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. When to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage.*

- *Pupils who grasp concepts rapidly should be challenged through rich and sophisticated problems before any acceleration through new content. Those pupils who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.*

For many schools and teachers the shift to this 'mastery curriculum' will be a significant one. It will require new approaches to lesson design, teaching, use of resources and support for pupils.

Key features of the mastery approach

Curriculum design

A detailed, structured curriculum is mapped out across all phases, ensuring continuity and supporting transition. Effective mastery curricula in mathematics are designed in relatively small carefully sequenced steps, which must each be mastered before pupils move to the next stage. Fundamental skills and knowledge are secured first. This often entails focusing on curriculum content in considerable depth at early stages.

Teaching resources

A coherent programme of high quality curriculum materials is used to support classroom teaching. Concrete and pictorial representations of mathematics are chosen carefully to help build procedural and conceptual knowledge together. Exercises are structured with great care to build deep conceptual knowledge alongside developing procedural fluency.

The focus is on the development of deep structural knowledge and the ability to make connections. Making connections in mathematics deepens knowledge of concepts and procedures, ensures what is learnt is sustained over time, and cuts down the time required to assimilate and master later concepts and techniques.

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One medium for coherent curriculum materials is high quality textbooks. These have the additional advantage that pupils also use them to return to topics studied, for consolidation and for revision. They represent an important link between school and home.

Lesson design

Lessons are crafted with similar care and are often perfected over time with input from other teachers, drawing on evidence from observations of pupils in class. Lesson designs set out in detail well-tested methods to teach a given mathematical topic. They include a variety of representations needed to introduce and explore a concept effectively and also set out related teacher explanations and questions to pupils.

Teaching methods

In highly successful systems, teachers are clear that their role is to teach in a precise way which makes it possible for all pupils to engage successfully with tasks at the expected level of challenge. Pupils work on the same tasks and engage in common discussions. Concepts are often explored together to make mathematical relationships explicit and strengthen pupils' understanding of mathematical connectivity.

Precise questioning during lessons ensures that pupils develop fluent technical proficiency and think deeply about the underpinning mathematical concepts. There is no prioritisation between technical proficiency and conceptual understanding; in successful classrooms these two key aspects of mathematical learning are developed in parallel.

Pupil support and differentiation

Taking a mastery approach, differentiation occurs in the *support and intervention provided* to different pupils, *not in the topics taught,* particularly at earlier stages. There is no differentiation in content taught, but the questioning and scaffolding individual pupils receive in class as they work through problems will differ, with higher attainers challenged through more demanding problems which deepen their knowledge of the same content. Pupils' difficulties and misconceptions are identified through immediate formative assessment and addressed with rapid intervention – commonly through individual or small group support later the same day: there are very few "closing the gap" strategies, because there are very few gaps to close.

Productivity and practice

Fluency comes from deep knowledge and practice. Pupils work hard and are productive. At early stages, explicit learning of multiplication tables is important in the journey towards fluency and contributes to quick and efficient mental calculation. Practice leads to other number facts becoming second nature. The ability to recall facts from long term memory and manipulate them to work out other facts is also important.

All tasks are chosen and sequenced carefully, offering appropriate variation in order to reveal the underlying mathematical structure to pupils. Both class work and homework provide this 'intelligent practice', which helps to develop deep and sustainable knowledge.

Implications for professional development and training of teachers

Teachers of mathematics in countries that perform well in international comparisons are mathematics specialists, including those in primary schools. They have deep subject knowledge, and deep knowledge of how to teach mathematics. They engage in collaborative planning and are continually seeking to improve their effectiveness.

Specialist mathematics teachers will therefore require:

- Deep structural subject knowledge of mathematics;
- Strong understanding of the structure of the curriculum and its aims: fluency, accuracy, precision, reasoning and problem solving, and how to apply these to teaching;
- Insight of what is meant by mastery of the curriculum, the factors that contribute to it and how it is achieved;
- Appreciation of the importance of practice and the nature of intelligent practice to develop deep and sustainable understanding which contributes to mastery for all;
- Effective strategies to support pupils to learn, recall and apply multiplication tables;
- Knowledge of mathematics as a network of interconnected ideas and an appreciation that making connections reduces the amount of mathematics to learn, deepens knowledge and contributes to sustainability of understanding over time;
- The ability to select and employ effectively the use of mathematical representations to enable pupils to access the underlying structure of the mathematics;
- An appreciation of the features of good textbooks and when and how to use them appropriately to support high quality teaching;
- Opportunities to collaborate with other professionals;
- Knowledge of how effectively to deliver high quality whole class teaching and provide access for all pupils; and

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- The ability to provide quick feedback to pupils and effective intervention to support all pupils to keep pace with the rest of the class.